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Optimizing Profit
Math 1010 Signature Assignment, Spring 2015

Group size is 3 to 4 students. One person is not a group and any projects submitted by individual students will not be graded.

Your group will submit one project. Be sure that all of your names are on it, and be sure that each member of the group approves of the final draft. Your work will be neat and legible. All mathematical calculations will be shown in a neat and logical progression.

Background Information:

Linear Programming is a technique used for optimization of a real-world situation. Examples of optimization include maximizing the number of items that can be manufactured or minimizing the cost of production. The equation that represents the quantity to be optimized is called the objective function, since the objective of the process is to optimize the value. In this project the objective is to maximize the profit of a small business.

The objective is subject to limitations or constraints that are represented by inequalities. Limitations on the number of items that can be produced, the number of hours that workers are available, and the amount of land a farmer has for crops are examples of constraints that can be represented using inequalities. Manufacturing an infinite number of items is not a realistic goal. In this project some of the constraints will be based on budget.

Graphing the system of inequalities given by the constraints provides a visual representation of the possible solutions to the problem. If the graph is a closed region, it can be shown that the values that optimize the objective function will occur at one of the "corners" of the region.

The Problem:

In this project your group will solve the following situation:

A small business produces collars and leashes for dogs. The monthly costs and budget have been calculated. The cost of materials for each collar is \$3.00 and the cost of materials for each leash is \$4.00. The cost of labor will amount to \$13.00 for each collar and \$6.00 for each leash. The business does not wish to spend more than \$240 on materials and \$496 on labor each month. In addition, they want to produce at least 50 items per month but no more than 100 items per month. The profit from each collar will be \$9.50, and the profit from each leash will be \$10.25. How many of each item should be made to obtain the maximum profit?

materials	Labor	Spending
Collar - \$3.00	collar - \$13.00	no more than 240
Leash - \$4.00	Leash - \$6.00	no more than 496

Modeling the Problem:

Let x be the number of dog collars that are made and y be the number of leashes that are made.

1. Write down a linear inequality that models how the materials costs will be kept within budget.

   $3x + 4y \leq 240$

2. Write down a linear inequality that models how the labor costs will be kept within budget.

   $13x + 6y \leq 496$

3. Recall that the business wants to produce at least 50 items each month. Write down a linear inequality to model this constraint.

   $x + y \geq 50$

4. The business wants to produce no more than 100 items each month. Write down a linear inequality to model this constraint.

   $x + y \leq 100$

5. There are two more constraints that must be met. These relate to the fact that it is impossible to manufacture a negative number of items. Write the two inequalities that model these constraints: $x \geq 0$

   $y \geq 0$

6. Next, write down the function for the profit that will be earned by selling the dog collars and leashes. This is the Objective Function for the problem.

   $P = 9.50x + 10.25y$

You now have six linear inequalities and an objective function. These together describe the situation. This combined set of inequalities and objective function make up what is known mathematically as a **linear programming** problem. Write all of the inequalities and the objective function together below. This is typically written as a list of constraints, with the objective function last.

1 $3x + 4x \leq 240$

2 $13x + 6x \leq 496$

3 $x + y \geq 5$

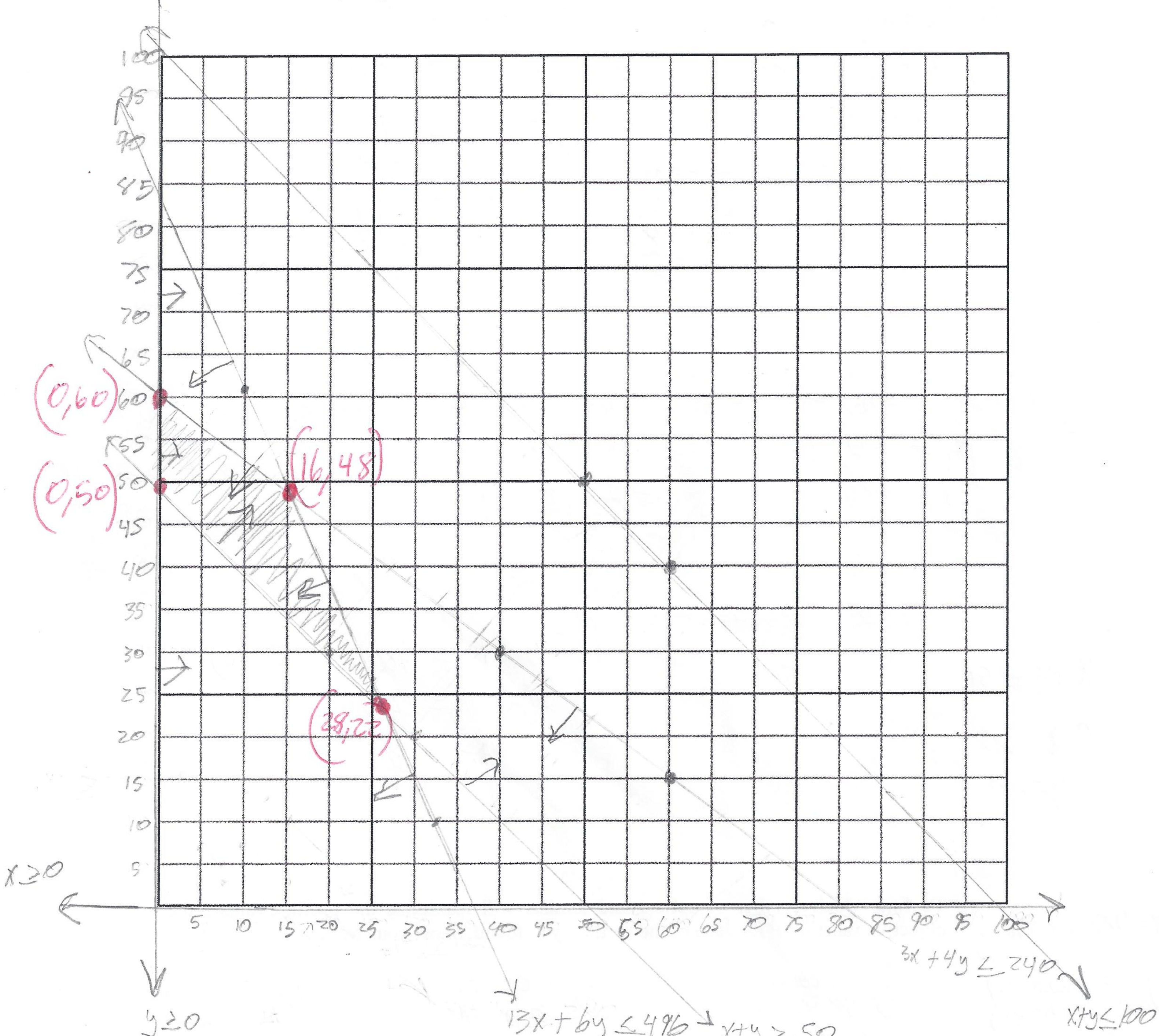
4 $x + y \leq 100$

5 $x \geq 0$

6 $y \geq 0$

$P = 9.50x + 10.25y$

7. To solve this problem, you will need to graph the **intersection** of all six inequalities on one common *x,y*-plane. Do this on the grid below. Have the bottom left be the origin, with the horizontal axis representing x and the vertical axis representing *y*. Label the axes with appropriate numbers and verbal descriptions, and label your lines as you graph them. You may use your own graph paper or graphing software if you prefer.



8. The shaded region in the above graph is called the <u>feasible region</u>. Any (*x*, *y*) point in the region corresponds to a possible number of collars and leashes that will meet all the requirements of the problem. However, the values that will maximize the profit will occur at one of the vertices or corners of the region. Your region should have four corners. Find the coordinates of these corners <u>by solving the appropriate system of linear equations</u>. Be sure to *show your work* and *label* the (*x*, *y*) coordinates of the corners in your graph.

1) $13x + 6y = 496$
2) $x + y = 50$
$x = 50 - y$

1) $13(50-y) + 6y = 496$
$650 - 13y + 6y = 496$
$650 - 7y = 496$
$-650 \quad -650$
$\frac{-7y}{-7} = \frac{-154}{-7}$
$y = 22$

Plug in: $x + 22 = 50$
$-22 \quad -22$
$x = 28$

(28, 22)

1) $13x + 6y = 496) \cdot -2$
2) $3x + 4y = 240) \, 3$

$-26x - 12y = -992$
$9x + 12y = 720$
$\frac{-17x}{-17} = \frac{-272}{-17}$
$x = 16$

(16, 48)

Plug in: $13(16) + 6y = 496$
$208 + 6y = 496$
$\frac{6y}{6} = \frac{288}{6}$
$y = 48$

$x + y = 50$
$0 + y = 50$
$y = 50$

(0, 50)

$3x + 4y = 240$
$3(0) + 4y = 240$
$\frac{4y}{4} = \frac{240}{4}$

(0, 60)

9. To find which number of collars and leashes will maximize the profit, evaluate the objective function *P* for each of the vertices you found. Show your work.

$P = 9.50x + 10.25y$

1) 28 collars (x)
22 leashes (y)
$9.50(28) + 10.25(22)$
$266 + 225.5$
$P = 491.5$

2) 16 collars
48 leashes
$9.50(16) + 10.25(48)$
$152 + 492$
$P = 644$

3) 0 collars
50 leashes
$9.50(0) + 10.25(50)$
$P = 512.5$

4) 0 collars
60 leashes
$9.50(0) + 10.25(60)$
$P = 615$

10. Write a sentence describing how many of each item should be manufactured to produce the highest profit. Include the amount of the profit that will be earned.

The company will need 16 collars and 48 leashes in order to obtain a maximum profit of $644.

GROUP PROJECT DUE:

Each group member will also need a scanned copy of the project to put in their e-Portfolio. Instructions for the reflective writing and e-Portfolio assignment are available on the class website.